Homework – 1

1. [20 points] Fill in all the missing values. For column A you need to compute the sums. For column B (the last two rows) you need to guess a function that does not contradict any of the yes/no answers already in the next three columns. Fill in each empty entry in the last three columns with either a yes/no answer.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Function | Function | O | Ω | θ |
| A | B | A = O(B) | A = Ω(B) | A = θ(B) |
| n4 | n3 lg n |  |  |  |
|  | n2 |  |  |  |
| (n + 1)! | n! |  |  |  |
| lg n | nk where k > 0 |  |  |  |
| å + = = n i i 1 ( 1) |  |  |  |  |
|  |  |  |  |  |

1. [20 points] Prove the following using the original definitions of O, Ω, θ, o and ω.
2. 3n3 + 50n2 + 4n - 9 ∈ O(n3)

Solution:

The definition of Big-Oh states that

g(n) ∈ O(f(n)): { g(n) : there exists positive constants c and N such that 0 ≤ g(n) ≤ c f(n) for all n ≥ N}

0 ≤ 3n3 + 50n2 + 4n – 9 ≤ cn3

3n3 + 50n3 + 4n3 – 9n3 ≤ cn3

57n3-9n3 ≤ cn3

48n3 ≤ cn3

For n≥ 1 the condition of Big-Oh satisfies. Hence, 3n3 + 50n2 + 4n - 9 ∈ O(n3)

1. 1000n3 ∈ Ω(n2)

Solution:

The definition of Omega states that

g(n) ∈ Ω (f(n)): { g(n) : there exists positive constants c and N such that 0 ≤ c f(n) ≤ g(n)for all n ≥ N}

1000n3 ≥ cn2

1000n2 ≥ cn

For n ≥ 1 and c ≤ 1000 the condition of Omega holds true.

(c) 10n3 +7n2 ∈ ω(n2)

(d) 78n3 ∈ o(n4)

(e) n2 + 3n – 10 ∈ θ(n2)